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The guided waves modelling - a spectral method approach.

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Guided elastic waves are used extensively in the nondestructive evaluation (NDE) of various structures. An important step in the development of inspection techniques is the accurate and efficient calculation of dispersion curves [2].

Traditionally, mode dispersion was studied by finding roots of analytical dispersion equations. For example, the thick cylindrical bar dispersion relation $f(x, \gamma a)$ for axisymmetric motion is defined as

$$\left(2 - x^2\right)^2 \mathrm{J}_0\left(\gamma a A\right) \mathrm{J}_1\left(\gamma a B\right) + 4AB\mathrm{J}_1\left(\gamma a A\right) \mathrm{J}_0\left(\gamma a B\right) - \frac{2x^2}{\gamma a} A\mathrm{J}_1\left(\gamma a A\right) \mathrm{J}_1\left(\gamma a B\right) = 0,$$

where a is radius of the semi-infinite bar, γ is wavenumber, x is the ratio of the phase velocity and the shear wave velocity, κ means the ratio of the squares of the phase velocities for the bar's material, $A = \sqrt{\kappa x^2 - 1}$, $B = \sqrt{x^2 - 1}$ and J is the Bessel function of the first kind.

The resulting dispersion curves are shown (for Poisson's number 0.3) in the Fig. 1 using solid gray lines.

An alternative approach to model two-dimensional circular structures was recently introduced by Adamou and Craster [1] based on spectral methods. The idea of this method is to solve the underlying differential equations by numerical interpolation using orthogonal polynomials and spectral differentiation matrices.

The spectral method approach for a free solid cylinder consists of the following steps:

1. The formulation of the underlying equations in cylindrical coordinates.
2. The formulation of the eigenvalue problem.
3. The solution of the eigenvalue problem for an elastic cylinder by means of the spectral method.

We will show the results of this approach in the form of dispersion curves. In Fig. 1 – black dots, the dispersion curves for a free solid cylindrical bar are computed for Poisson's number $\mu = 0.3$ and number of discretization points $N = 60$. These curves (dots) are in excellent agreement with the dispersion curves, which were calculated analytically using *root-finding* techniques. The only defect in beauty is low frequency behaviour that is caused by the singularity in cylindrical coordinates $(1/r)$. In the cylinder, you need to drill a small hole.

Traditional techniques require finding complex roots of nonlinear equations that involve special functions. In contrast, spectral method demands only solving generalized eigenvalue problem without involving special functions.

For numerical experiments were used MATLAB's toolbox CHEBFUN [3] and Julia's package ApproxFun [4].

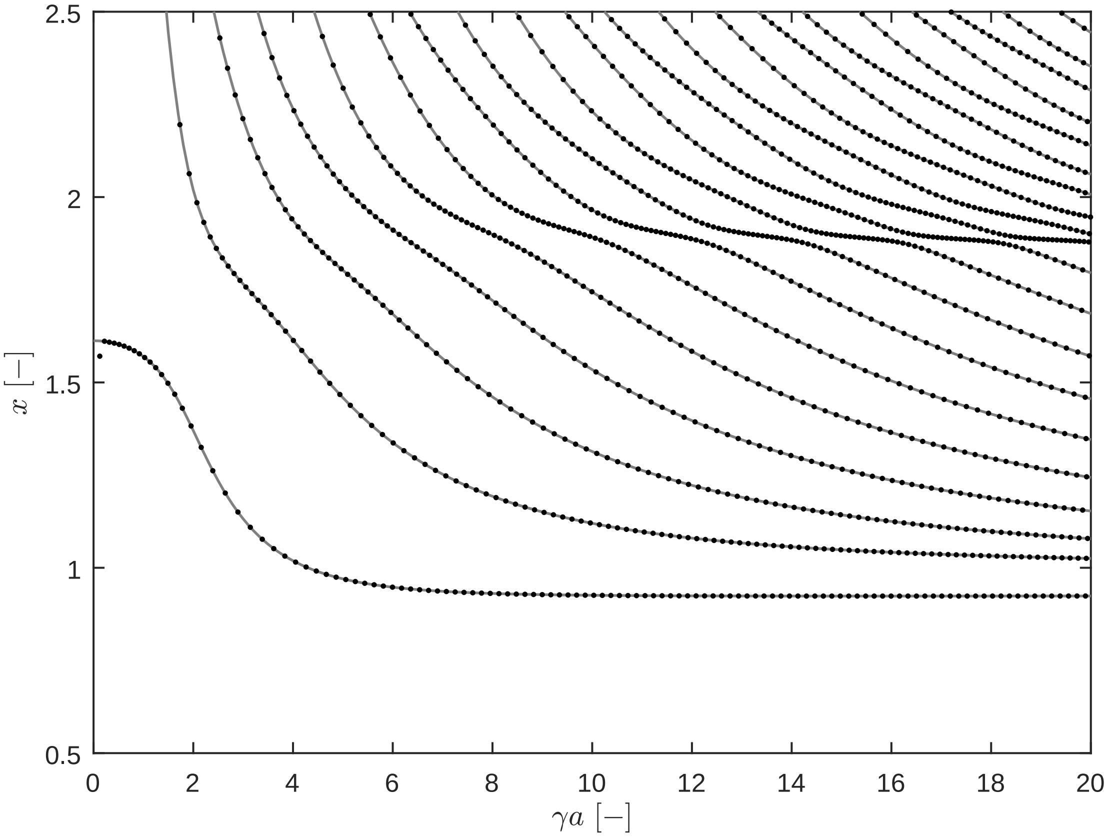

Fig. 1. Dispersion curves: analytical method (gray solid) and spectral method (black dots).

Acknowledgements

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References

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[3] Driscoll, T.A., Hale, N., Trefethen, L.N., editors, Chebfun Guide, Pafnuty Publications, Oxford, 2014.

[4] URL: github.com/ApproxFun/ApproxFun.jl.git