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The use of the Chebyshev interpolation in elastodynamics problems.

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For the analytical solution of the thick bar impact problem it is needed to evaluate a complicated oscillating integrals for displacements and stresses. The integration along the dispersion curve is generally calculated by means of trapezoid or Simpson's rule numerical integration (i.e. methods with equally spaced points).

This dispersion relation $f(x, \gamma a)$ (see Fig. 1 left) is defined as

$$\left(2-x^2\right)^2 \mathrm{J}_0\left(\gamma a A\right) \mathrm{J}_1\left(\gamma a B\right)+4 A B \mathrm{J}_1\left(\gamma a A\right) \mathrm{J}_0\left(\gamma a B\right)-\frac{2 x^2}{\gamma a} A \mathrm{J}_1\left(\gamma a A\right) \mathrm{J}_1\left(\gamma a B\right)=0,$$

where a is radius of the semi-infinite bar, γ is wavenumber, x is the ratio of the phase velocity and the shear wave velocity, κ means the ratio of the squares of the phase velocities for the bar's material, $A=\sqrt{\kappa x^2-1}$, $B=\sqrt{x^2-1}$ and J is the Bessel function of the first kind.

For the purpose of the speed up calculation, the dispersion curves are precalculated in equidistant points of γa.

The disadvantages of this process are: 1. impossibility to integrate from zero, 2. problematic choice of γa-step.

In order to remove the first restriction, another form of the dispersion relations is used. After making the substitution, $z = x\gamma a$, the dispersion relation $g(z, \gamma a)$ (see Fig. 1 right) is defined as

$$\left(2\gamma a^2-z^2\right)^2 \mathrm{J}_0\left(A\right) \mathrm{J}_1\left(B\right)+4\gamma a^2 A B \mathrm{J}_1\left(A\right) \mathrm{J}_0\left(B\right)-2z^2 A \mathrm{J}_1\left(A\right) \mathrm{J}_1\left(B\right)=0,$$

where $A=\sqrt{\kappa z^2-\gamma a^2}$ and $B=\sqrt{z^2-\gamma a^2}$.

In order to remove the second restriction, the integration method with unequally spaced points is used. For non-periodic functions, methods with unequally spaced points such as Gaussian quadrature and Clenshaw-Curtis quadrature are generally far more accurate. For using of these integration methods, the dispersion curves were approximated by the Chebyshev polynomials [2].

In Table 1 the properties of Chebyshev approximation for given curves are shown (vertical scale, total length of polynomial). For comparison, 100 curves for γa from 0.001 to 100 with step 0.001 take the 80MB file. 100 curves for γa from **0** to 100 created by the Chebyshev approximation take only the 1.3MB file.

While using the Gaussian or Clenshaw-Curtis quadrature, the calculation time has provided a 100x speedup over the original.

For numerical experiments were used MATLAB's toolbox CHEBFUN [1] and Julia's package ApproxFun [3].

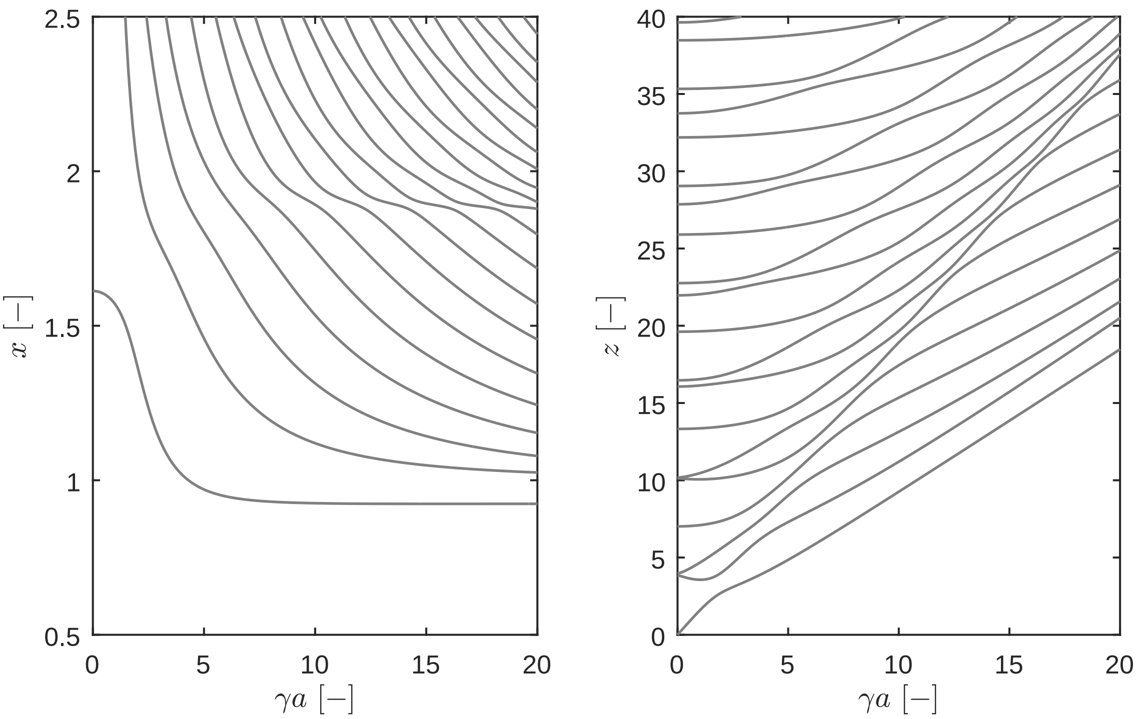

Fig. 1. Dispersion curves: $x - \gamma a$ form (left) and $z - \gamma a$ form (right).

Tab. 1. The Chebyshev approximation properties for any dispersion curves.

Curve #	Vertical scale	Length (splitting=off)	Length (splitting=on)
1	93	396	168
2	100	454	218
3	100	455	209
10	100	1821	301
20	120	4226	597
30	140	6527	729

Acknowledgements

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References

[1] Driscoll, T.A., Hale, N., Trefethen, L.N., editors, Chebfun Guide, Pafnuty Publications, Oxford, 2014.

[2] Snyder, M.A., Chebyshev Methods in Numerical Approximation, Prentice-Hall, 1966

[3] URL: github.com/ApproxFun/ApproxFun.jl.git