IDENTIFICATION OF CRITICAL SPEEDS USING UNBALANCE RESPONSE DATA

M. Balda [1]

1. Introduction

There are many cases in mechanical engineering, where a structure under observation is excited by centrifugal forces. The excitation itself is then generated by rotating unbalance masses coming either from random residual eccentricity or from deterministic eccentric weights in case of inertial actuators. The first case, typical for unbalanced rotors, is the subject of interest within this contribution.

Collected data contain speed n [rpm] and parameters of vibration, either deviations $q(t)$ or velocities $\dot{q}(t)$ and or accelerations $\ddot{q}(t)$. Let us assume that it is possible to approximate the rotor by a linear discrete system, and that we measure deviations $q(t)$. There are two ways of expressing the total information on vibration. Both describe the deviations as complex numbers

$$q = \mathrm{Re}\, q + i\, \mathrm{Im}\, q \qquad \text{or} \qquad q = |\, q \,|\, \mathrm{e}^{i\varphi} \quad \text{with} \quad \varphi = \mathrm{arctg}\, \frac{\mathrm{Im}\, q}{\mathrm{Re}\, q}$$

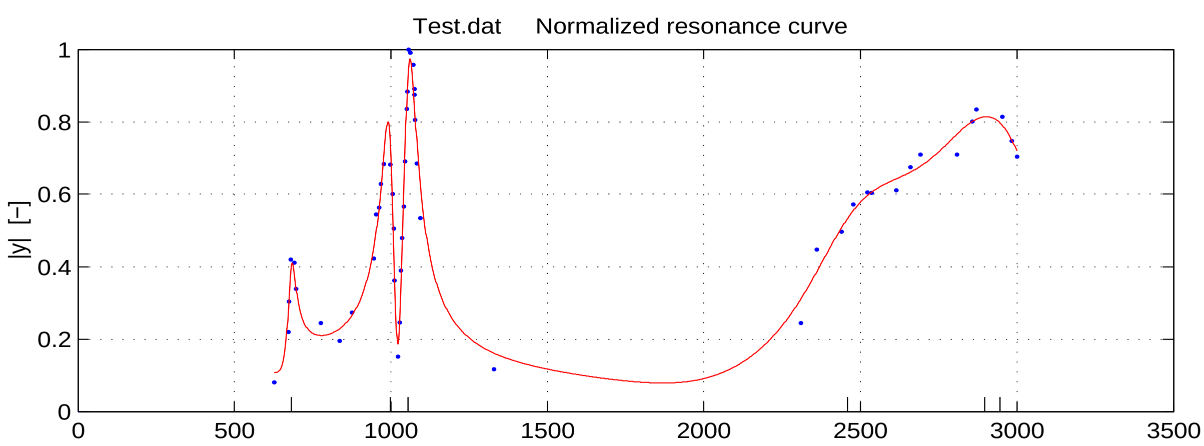

Figure 1: Normalized amplitudes of vibrations of a bearing housing during run-down of a machine

Usually, critical speeds are estimated from the peaks of plotted resonance curves, which are courses of amplitudes of vibrations $|\, q(p) \,|$ as functions of speed. This approach possesses

[1] Prof. Ing. Miroslav BALDA, DrSc., New Technologies Research Center of the West Bohemian Region, Univerzitní, 306 14 Plzeň, The Czech Republic, phone: 019-7236415, fax: 019-7220787, balda@cdm.it.cas.cz

many drawbacks consisting in poor accuracy of both frequency and damping estimates. It is seen from figure 1 that the estimation of critical speeds is difficult, if they are gathered in clusters. The contribution shows how to reach better estimates of both quantities from vibration measurements at a single point of a machine without any knowledge of exciting forces.

2. Theory

The motion of a discrete linear mechanical system with many degrees of freedom behaves under the equation

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}}(t) + \boldsymbol{B}\,\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\,\boldsymbol{q}(t) = \boldsymbol{f}(t) \tag{1}$$

It has been described elsewhere (see [2]) that the Fourier transform of a general response $\boldsymbol{q}(t)$ of the system excited by a general set of forces $\boldsymbol{f}(t)$ takes for $p = i\omega = i2\pi f$ the form

$$\boldsymbol{q}(p) = \boldsymbol{G}(p)\,\boldsymbol{f}(p) \tag{2}$$

with a matrix of frequency responses

$$\begin{aligned} \boldsymbol{G}(p) &= \left[p^2\,\boldsymbol{M} + p\,\boldsymbol{B} + \boldsymbol{K}\right]^{-1} \\ &= \boldsymbol{V}_q\,[\,p\,\boldsymbol{I} - \boldsymbol{S}\,]^{-1}\,\boldsymbol{W}_q^H \end{aligned} \tag{3}$$

Matrices $\boldsymbol{V}_q$ and $\boldsymbol{W}_q$ are built out of deviation parts of modal matrices which are composed out of left and right eigenvectors respectively. Matrix $\boldsymbol{S}$ is a diagonal spectral matrix of the problem possessing eigenvalues, scaled critical speeds, of the system under observation.

As soon as the excitation is generated by unbalanced masses rotating with p_k, the Fourier transforms of forces and corresponding responses are

$$\boldsymbol{f}(p_k) = -p_k^2\,\delta(p - p_k)\,\boldsymbol{u} \qquad \text{and} \qquad \boldsymbol{q}(p_k)\,\delta(p - p_k)\,, \tag{4}$$

respectively. The symbol $\delta(p - p_k)$ is the Dirac impuls in the frequency domain at p_k. A complex unbalance vector $\boldsymbol{u}$ is composed of static moments $m_j\,r_j$ of unbalanced masses m_j. Hence, the equation (2) may be rewritten with the use of equations (3) and (4) into the form

$$\boldsymbol{q}(p) = -\boldsymbol{V}_q\,\left[p^2\,(p\,\boldsymbol{I} - \boldsymbol{S})^{-1}\right]\,\boldsymbol{W}_q^H\,\boldsymbol{u} \tag{5}$$

Should the matrices $\boldsymbol{S}$, $\boldsymbol{V}_q$, $\boldsymbol{W}_q$ and a distribution of unbalances $\boldsymbol{u}$ along the rotor be known, the responses $\boldsymbol{q}(p)$ might be obtained for every p. However, this is not the case as usual. The matrices $\boldsymbol{S}$, $\boldsymbol{V}_q$, $\boldsymbol{W}_q$ are unknown and could be obtained out of measured responses $\boldsymbol{q}_j(p)$ and corresponding known set of unbalances $\boldsymbol{u}_j$.

Another situation raises up, when there are only one initial unknown unbalance vector $\boldsymbol{u}$ and vector of measured responses $\boldsymbol{q}(p)$ for different speeds at disposal. An unbalanced rotor may be considered as a single input system, because the unbalances $\boldsymbol{u}$ remain constant. Should the rotor be equipped by a set of pickups for measuring its movement, the whole system might be considered as a Single-Input-Multiple-Output one (SIMO system).

The identification of the rotor as a SIMO system may not be performed in one step because of the strong sensitivity of the identification proces to the measuring noise. At first, it is necessary to filter out all noise at every single point of measurement. It may be done by processing data belonging to that point via identification a SISO (Single-Input-Single-Output) system. This first step of identification plays very important role in the process.

It excerpts the basic properties of the system and serves as a "modal filter". This simpler identifications yields eigenvalues of the system observable at the measuring point.

For the purpose, the i-th element $q_i(p)$ of the deviation vector $\boldsymbol{q}(\mathrm{p})$ is possible to express from equation (5) as

$$q_i(p) = -\sum_{\nu} \frac{p^2}{p - s_\nu} \, \{\boldsymbol{v}_i^T\}_\nu \{\boldsymbol{W}_q^H \boldsymbol{u}\}_\nu \tag{6}$$

The expressions $\{\boldsymbol{v}_i^T\}_\nu$ and $\{\boldsymbol{W}_q^H \boldsymbol{u}\}_\nu$ denote scalars, the ν-th elements of the vectors $\boldsymbol{v}^T$, a row of the matrix $\boldsymbol{V}_q$, and the column vector $\boldsymbol{W}_q^H \boldsymbol{u}$, respectively. The negative product of both scalars is a scalar $a_{i\nu}$. Thus we are faced the nonlinear problem of finding the unknowns $a_{i\nu}$ and s_ν out of a set of observations taken at frequencies p_k

$$q_i(p_k) = \sum_{\nu} \frac{a_{i\nu}\, p_k^2}{p_k - s_\nu} \tag{7}$$

A procedure, similar to that described in [1], may be used for solving the problem. In order to diminish an influence of modes not included to the identification, it is possible to introduce a correction to $q_i(p_k)$. If it were linear in p, the form of corrected $q_i(p_k)$ would be

$$q_i(p_k) = \sum_{\nu} \frac{a_{i\nu}\, p_k^2}{p_k - s_\nu} + h_1 + h_2\, p_k \tag{8}$$

The quantities h_1 and h_2 are new unknown complex coefficients to be determined during the identification process. It is based on the Newton-Raphson optimization method for minimizing a sum of squares of residuals $S = \boldsymbol{r}^H \boldsymbol{r}$, where the k-th residual is a difference between the approximation $q_i(p_k)$ and measured $q_{mi}(p_k)$

$$r_k = \sum_{\nu} \frac{a_{i\nu}\, p_k^2}{p_k - s_\nu} + h_1 + h_2\, p_k - q_{mi}(p_k) \tag{9}$$

The method solves the problem in iterations. The result is updated in the step ℓ due to the formula

$$\boldsymbol{x}_i^{(\ell+1)} = \boldsymbol{x}_i^{(\ell)} - \left[\boldsymbol{J}^{(\ell)}\right]^+ \boldsymbol{r}^{(\ell)} \tag{10}$$

The vector of unknowns $\boldsymbol{x}_i$ is composed out of sub-vectors of eigenvalues $\boldsymbol{s} = [\, s_\nu]$, sensitivities $\boldsymbol{a}_i = [\, a_{i\nu}]$ and coefficients of corrections $\boldsymbol{h} = [\, h_1,\, h_2]^T$, creating thus a column vector of unknowns $x_i^T = [\, \boldsymbol{s}^T,\ \boldsymbol{a}_i^T,\ \boldsymbol{h}^T]$. Hence, the Jacobi matrix $\boldsymbol{J} = \partial S / \partial \boldsymbol{x}$ has the form

$$\boldsymbol{J} = \left[\frac{\partial S}{\partial \boldsymbol{s}},\, \frac{\partial S}{\partial \boldsymbol{a}},\, \frac{\partial S}{\partial \boldsymbol{h}} \right] = [\, \boldsymbol{A}_2,\, \boldsymbol{A}_1,\, \mathbf{1},\, \boldsymbol{p}\,]\ , \tag{11}$$

where $\mathbf{1}$ is a column vector of all ones. The submatrices $\boldsymbol{A}_j$ have, for row index k of frequencies and column index ν of modes, the form

$$\boldsymbol{A}_1 = \begin{bmatrix} & \vdots & \\ \cdots & \dfrac{p_k^2}{p_k - s_\nu} & \cdots \\ & \vdots & \end{bmatrix} \quad \text{and} \quad \boldsymbol{A}_2 = \begin{bmatrix} & \vdots & \\ \cdots & \dfrac{a_\nu\, p_k^2}{(p_k - s_\nu)^2} & \cdots \\ & \vdots & \end{bmatrix}. \tag{12}$$

3. Programming and test results

The program for solving the given problem has been built in the programming language MATLAB, which enables to solve dynamical problems rather efficiently. The program is composed out of following modules – M-functions:

critspeed	main driver, for I/O and calling working procedures
idecrit	identification procedure calling MATLAB Optimization Toolbox function lsqnonlin performing Newton-Raphson algorithm for minimizing goal function $S(\boldsymbol{x})$,
peaks	function for finding peaks of resonance curves $\mid y(p) \mid$ necessary for initial estimate of critical speeds of a rotor
funJ	function, required by lsqnonlin, providing a vector of weighted residuals and the Jacobi matrix $\boldsymbol{J}$
qp	function for evaluating $q(\boldsymbol{p})$ for arbitrary $\boldsymbol{p}$ using known $\boldsymbol{s}$, $\boldsymbol{a}$, $\boldsymbol{h}$
tick	function for drawing ticks along speed axis in places of critical speeds

The program is interactive. A user may choose certain parameters, which influence tasks and their solution. The measurement comes in the form of an ASCII-file, the name of which is input by the user from a keyboard. The file is filled by a table containing arbitrary number of rows composed of triples of real numbers, speed in revolutions per minute and real and imaginary parts of the measured response of the rotor at a measuring point (see right). The input values of vibrations $q(p)$ are normalized to $y(p)$ the maximum amplitude of which equals one. These were plotted in figure 1 as points. The continuous line shows the identified response of the machine at the point of measurement.

rpm	Re q	Im q
628	2.0	-7.5
672	5.0	-20.5
675	3.8	-29.0
680	-10.5	-39.0
692	-24.5	-31.0
697	-24.5	-21.5
777	-19.5	-13.0
836	-18.0	-5.0
876	-26.0	3.5
946	-39.0	11.5
953	-47.0	23.0
962	-43.0	33.0
969	-43.5	42.0
977	-36.0	55.0
999	18.0	63.0
1005	27.0	51.0
1009	29.0	39.0
1012	23.0	26.0
1022	1.0	14.5
1028	-14.0	19.0
1032	-21.0	31.0
1036	-21.0	41.0
1042	-19.0	51.0
1046	-21.0	63.0
1050	-7.5	80.0
1052	1.5	85.0
1056	28.0	92.0
1062	43.0	85.0
1072	58.5	71.0
1075	64.0	57.0
1076	71.0	45.0
1078	69.0	35.0
1083	62.0	22.0
1095	50.0	12.0
1330	10.0	-5.0
2310	23.0	5.0
2360	43.0	0.0
2440	47.0	-8.0
2478	52.0	-18.0
2522	52.0	-26.0
2535	47.0	-34.0
2614	41.0	-42.0
2659	40.0	-51.0
2691	36.0	-58.0
2808	12.5	-67.0
2857	1.0	-77.0
2871	-5.0	-80.0
2953	-33.0	-71.0
2983	-38.0	-61.0
3000	-42.0	-53.0

The user may change default uniform weights of measurements into arbitrary function of frequency just like a setting of initial estimates of critical speeds rather simply. They may be found automatically as the speeds corresponding to peaks of the response, or input manually from the keyboard.

A problem of identification eigenvalues is solved in two levels. At the beginning, the initial estimates of $\boldsymbol{s}$ and $\boldsymbol{a}$ are sought by a method described in literature [2]. As soon as they are known, the lsqnonlin function is called, which minimizes sum of squares of residuals using function funJ which supplies residuals and Jacobi matrix. The optimization procedure yields the optimal values of natural frequencies $\boldsymbol{s}$ and their intensities $\boldsymbol{a}$ at the point of observation and correcting factors $\boldsymbol{h}$. Simultaneously, the identified normalized response is output.

The output quantities serve for calculations of the relative dampings and Q-factors for every mode, critical speed, using formulae

$$b_{p\nu} = \frac{-\mathrm{Re}\ s_\nu}{\mid s_\nu \mid}\ , \qquad\qquad Q_\nu = \frac{1}{2\, b_{p\nu}} \tag{13}$$

These values are very important, because they inform the user on the degree of danger of the particular critical speed.

The results are both numerical and graphical and have the forms:

```
==================================================
   Critical speed identification    06-Aug-2001
==================================================

          file = Test.dat =>
          weight = ones(size(f)) =>

          use peaks = yes => n
          f peaks = [] => [685,1000,1050,2400,2900]

 Initial estimates of frequencies

   f( 1) = 11.333 [Hz] =  680.0 [ot/min]
   f( 2) = 16.650 [Hz] =  999.0 [ot/min]
   f( 3) = 17.500 [Hz] = 1050.0 [ot/min]
   f( 4) = 39.333 [Hz] = 2360.0 [ot/min]
   f( 5) = 47.850 [Hz] = 2871.0 [ot/min]

  Frequencies found by optimization

mode    Re n   -Im n      bp        Q     alpha

  1    681.4    13.0   0.0191   26.21  -175.1
  2    998.6    19.9   0.0199   25.11   -42.3
  3   1055.1    22.2   0.0210   23.78     4.5
  4   2458.3   214.5   0.0869    5.75   -83.7
  5   2897.0   147.8   0.0509    9.81    98.5
  6   2945.4   136.5   0.0463   10.80   178.4

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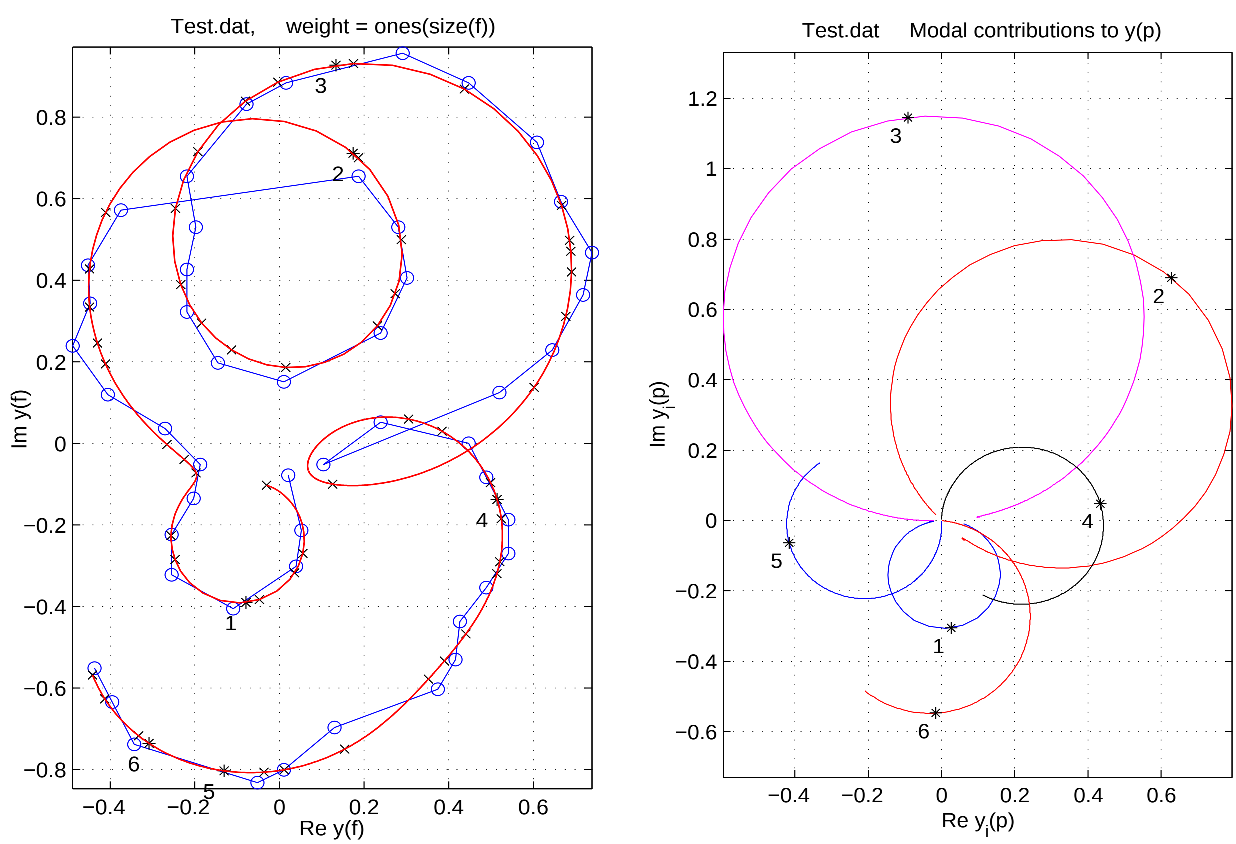

Figure 2: a. Normalized bearing vibrations due to an unknown unbalance
b. Modal components of normalized bearing vibrations
o - measured, x - identified, $*$ - eigenvalues; $y = |q|/\max|q|$

The figure 1 is also one of outputs of the programm run. The figure 2a contains both points of measured normalized vibrations marked by circles and connected by straight lines, and identified course plotted by a smooth line. The points of critical speeds are plotted for $\omega_\nu = \mathrm{Re}\, s_\nu$, numbered and marked by asterisks.

The decompositions of the total normalized response into its modal components is shown in the figure 2b for the whole interval of running speeds. The points of the critical speeds are again marked by asterisks. They also denote angles of 90° phases with respect to the phase-shift sensor. Angular positions of modal counterweights are again by 90° shifted with respect to 90° phase positions. The angles of counterweight modal planes are printed as last columns in the table of numerical results.

4. Conclusions

The presented method of identification critical speeds out of data obtained by measurement of vibration parameters as functions of rotor speed proved to be effective. It is applicable for precise estimation of critical speeds, dampings and positions of modal components of unbalances out of measurement taken in a single point of a machine, which is excited by unknown unbalance. The measurement point should be selected as a such one, which gives maximum information on dynamic properties of the rotor.

The measured response is possible to decompose into modal responses. The phase angles corresponding the points of eigenvalues on the modal responses serve for estimating the planes of modal unbalances, a knowledge of which may accelerate a balancing procedure.

More information could be obtained by simultaneous measurements in more points of the rotor. The present method could be used as a starting one for SIMO identification by a more complicated method similar to that presented in [2].

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IDENTIFICATION OF CRITICAL SPEEDS USING UNBALANCE RESPONSE DATA

A new method of the identification critical speeds out of noisy measurements of rotor responses to an unknown unbalance is described. The identification procedure is based on the fitting experimental data by a response of a Single-Input Single-Output (SISO) system. The method yields natural frequencies with dampings of the rotor out of data measured at a certain point of a machine. The result may serve to estimating planes of modal components of an unbalance. The SISO system identification has been implemented in MATLAB.