A method for local identification of critical speeds of rotors

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Critical speeds of rotors belong to the most persued phenomena in dynamics of rotors. It is understandable, because any enlarging amplitudes of vibrations leads to lowering a service life of machines due to potential fatigue cracks on machine parts.

Let us assume a rotor as a linear system being discretized into many degrees of freedom. Hence, its behaviour is described by a well known set of linear differential equations, which written in a matrix notation take the form

$$\boldsymbol{M}\,\ddot{\boldsymbol{q}}(t) + \boldsymbol{B}\,\dot{\boldsymbol{q}}(t) + \boldsymbol{K}\,\boldsymbol{q}(t) = \boldsymbol{f}(t) \tag{1}$$

There, $\boldsymbol{M}$ is a mass matrix, $\boldsymbol{B}$ a damping matrix and $\boldsymbol{K}$ a stiffness matrix. The vectors $\boldsymbol{q}$ and $\boldsymbol{f}$ contain generalized coordinates and forces, respectively. The frequency response matrix of the system has the form

$$\boldsymbol{G}(p) = \boldsymbol{V}_q\,[\,p\,\boldsymbol{I}_{2n} - \boldsymbol{S}\,]^{-1}\,\boldsymbol{W}_q^H \tag{2}$$

The symbols used in (2) mean: $p = i\omega$, $\boldsymbol{V}_q$ and $\boldsymbol{W}_q$ are the deflection parts of modal matrices of the right and left eigen-vectors, and $\boldsymbol{S}$ a spectral matrix. Fourier transforms $\boldsymbol{q}(p)$ of the rotor responses $\boldsymbol{q}(t)$ depend on an excitation by unbalance $\boldsymbol{u} = [m_j\,r_j]$ as described by the equation (3)

$$\begin{aligned} \boldsymbol{q}(p) &= \boldsymbol{G}(p)\,\boldsymbol{f}(p) \;=\; p^2\,\boldsymbol{G}(p)\,\boldsymbol{u} \\ &= \boldsymbol{V}_q\,p^2\,[\,p\,\boldsymbol{I}_{2n} - \boldsymbol{S}\,]^{-1}\,\boldsymbol{W}_q^H\,\boldsymbol{u} \end{aligned} \tag{3}$$

It is evident that there are very many unknowns in the equation influencing the responses $\boldsymbol{q}(t)$. Should the response be measured at a single point of the system, it would be described by the equation

$$\begin{aligned} q_i(p) &= \boldsymbol{v}_{qi}^T\,p^2\,[\,p\,\boldsymbol{I}_{2n} - \boldsymbol{S}\,]^{-1}\,\{\,\boldsymbol{W}_q^H\,\boldsymbol{u}\,\} \\ &= \sum_{\nu=1}^{n} \frac{p^2}{p - s_\nu}\,a_{\nu i} \end{aligned} \tag{4}$$

Quantities $\{s_\nu\}$ and $\{a_{\nu i}\}$ are to be identified. The function `lsqnonlin` of the Optimization Toolbox of MATLAB was used for minimizing a length of a vector of residuals $\boldsymbol{r}$ as a difference of measured responses $\boldsymbol{q}_m(p)$ and approximated $\boldsymbol{q}(p)$ under (4).

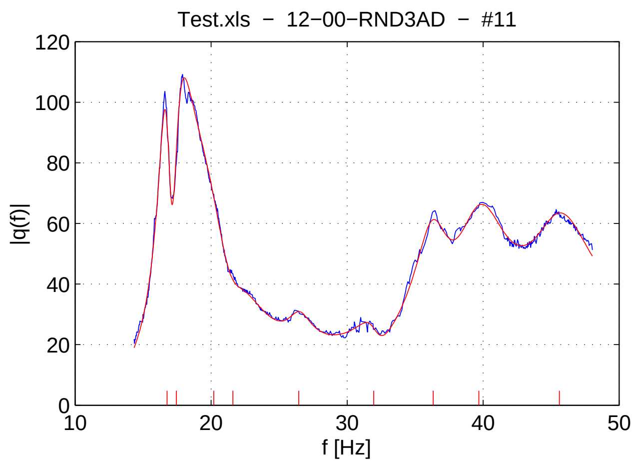
The speed of iteration process is maximal, if the Jacobian matrices $\boldsymbol{J}_s$ and $\boldsymbol{J}_a$ of the problem were known. Their elements are similar to those in [1]:

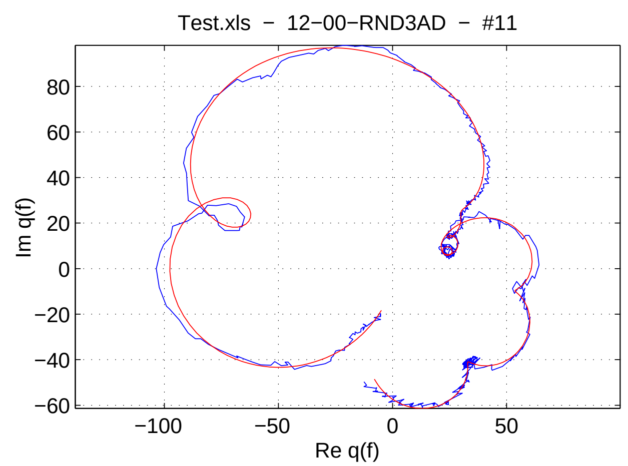
$$j_{\kappa\nu s} = \frac{p_\kappa^2\,a_\nu}{(p_\kappa - s_\nu)^2} \quad \text{and} \quad j_{\kappa\nu a} = \frac{p_\kappa^2}{p_\kappa - s_\nu} \tag{5}$$

The subscript κ belongs to the speed for which vibrations have been measured.

The solution is iterative, since the expressions in (5) contain unknowns on the right-hand sides. It is necessary to preset good estimates of unknowns at the beginning of the process. The identification procedure evaluates them from the measured responses before the iterations start.

The figure shows a result of processing a response to an unknown unbalance using just described procedure. The uneven line connects the values of vibration measured at one bearing during a run-down of a turboset. The smooth line is the identified one. The fit is quite good.





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References

[1] Kozánek J.: Parameter evaluation of a transfer function from measured data (In Czech). PhD thesis, IT ASCR, Prague, 1980