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FATIGUE DAMAGE OF BLADES UNDER TRANSIENTS

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Abstract: *The contribution deals with damaging blades of turbines caused by a sudden change of a loading. Such a change causes transients in vibrations of blades, and in consequence of it a cumulation of the fatigue damage. Four analytical models are studied.*

Key words: *blades, transients, fatigue damage*

1. INTRODUCTION

Blades of turbomachines are very important elements, which influence both efficiency and reliability of machines in very important way. No doubts that teams of specialists are working on problems of blades all over the world. Let us present a part of the study, which deals with a small, nevertheless very important problem of a fatigue damaging of blades caused by sudden changes of the turbine loading. In consequence of it, transient vibrations occur, mostly of the basic mode. They may generate high dynamic stress in critical places of the blades resulting in cumulating a fatigue damage.

2. MODELS OF TRANSIENS

Let us assume the blades are slender, slightly damped bars with a dominant influence of basic modes on the response to the external actions. It is very impossible to define an exact response of the blade, because it depends on the form and intensity of the above mentioned change of operational conditions. A different situation takes place after the shut down of a machine compared with that, which occurs during many kinds of short circuits on the electrical side of the system. This is the reason, why we have studied behaviours of several response models.

Model 1

A stress state in the critical place of a blade as a time function describing free vibrations with a frequency ω_o may be approximated by a differential equation corresponding a single degree of freedom system:

$$\ddot{\sigma}(t) + 2b_p\omega_o\dot{\sigma}(t) + \omega_o^2\sigma(t) = 0, \tag{1}$$

with b_p as a relative damping of the system. The solution of it is

$$\sigma(t) = \sigma_o e^{\lambda t}, \tag{2}$$

with σ_o as initial maximum value at $t = 0$, λ is an eigenvalue of the problem, which may be expressed as

$$\lambda_{1,2} = (-\beta \pm i)\omega_d \tag{3}$$

The frequency of damped vibration is ω_d, for which it holds

$$\omega_d = \omega_o\sqrt{1-b_p^2} \qquad \text{and} \qquad \beta = \frac{b_p}{\sqrt{1-b_p^2}} \tag{4}$$

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The stress in the critical position will change under the law

$$\sigma(t) = \sigma_o e^{\omega_d(-\beta \pm i)t} \tag{5}$$

A fatigue damage of a blade material is not determined by the whole course of the stress function of time, but by its extrema. The necessary condition for an existence of an extreme is a zero derivatives of $\sigma(t)$

$$\sigma_o \operatorname{Re}[\omega_d(-\beta \pm i) e^{\omega_d(-\beta \pm i)t}] = 0 \tag{6}$$

Hence, the extrema (maxima & minima) are coming in times

$$t_\mu = \frac{1}{\omega_d}\left(\mu\pi - \operatorname{arctg}\beta\right) \tag{7}$$

The maxima occur at every second time t_μ giving thus the sequence

$$\sigma_\nu \approx \sigma_o e^{-2\nu\pi\beta} \tag{8}$$

A ratio of two consecutive extremes possessing the same sign will be

$$\frac{\sigma_\nu}{\sigma_{\nu+1}} = e^{2\pi\beta} \qquad \Rightarrow \qquad \vartheta = \ln\frac{\sigma_\nu}{\sigma_{\nu+1}} = 2\pi\beta \tag{9}$$

The symbol ϑ is the well known logarithmic decrement, which is often used for expressing properties of materials of blades.

A fatigue damage caused by a transient vibrations may be estimated by means of the Palmgren-Miner hypothesis, which says that an elementary damage d_ν produced by ν-th stress cycle of the amplitude $\sigma_{a\nu}$ is

$$d_\nu = \frac{1}{N_\nu} = \frac{1}{N_c}\left(\frac{\sigma_{a\nu}}{\sigma_c}\right)^w, \tag{10}$$

with a number N_ν of stress cycles of the amplitude $\sigma_{a\nu}$ to the break, the fatigue limit σ_c of the material used, a corresponding number N_c of cycles, and a power w of the S-N curve.

If we neglect a decrease of vibration in the cycle, we may put $\sigma_{a\nu} \approx \sigma_\nu$., and the total damage of one single transient process of damped vibration express as

$$D = \sum_{\nu=0}^{n-1} d_\nu = \frac{1}{N_c \sigma_c^w}\sum_{\nu=0}^{n-1}\sigma_{a\nu}^w = \frac{1}{N_c}\left(\frac{\sigma_o}{\sigma_c}\right)^w \sum_{\nu=0}^{n-1} e^{-2\pi\nu\beta w} = \frac{1}{N_c}\left(\frac{\sigma_o}{\sigma_c}\right)^w \sum_{\nu=0}^{n-1} e^{-\vartheta\beta w}, \tag{11}$$

with the number n of cycles with damaging effect. The expression behind the summation sign is a geometric series with the first element $a_1 = 1$ and a quotient $q = e^{-\vartheta w}$. The total of this series equals

$$s_n = \frac{a_1\left(q^n - 1\right)}{q - 1} = \frac{e^{-n\vartheta w} - 1}{e^{-\vartheta w} - 1} \tag{12}$$

The total damage D caused by a single transient process becomes

$$D = \frac{1}{N_c}\left(\frac{\sigma_o}{\sigma_c}\right)^w s_n, \tag{13}$$

with a number n of damaging cycles coming from the Palmgren-Miner hypothesis for $\sigma_{a\nu} \in \langle \sigma_c, \sigma_o \rangle$ from

$$\frac{\sigma_o}{\sigma_c} = e^{n\vartheta} \qquad \Rightarrow \qquad n = \operatorname{int}\frac{\ln\sigma_o - \ln\sigma_c}{\vartheta} \tag{14}$$

Hence, the damage of the model 1 may be estimated rather easily due to the closed formula (13).

Model 2

Measurements carried out on real blades have revealed that transients generated after a step change of power are different from the model 1. Pressure distribution in the machine initiated by the drop of a load is changing in time influencing thus a raise of turbulence, which excites blades. The formula

$$\sigma(t) = \sigma_o t^{\alpha} e^{(-\beta \pm i)\omega_d t} \tag{15}$$

seemed to be a good approximation of the real stress process. Extremes take place at times

$$t_{\nu} = \frac{1}{\omega_d}\left[\nu\pi - \operatorname{arctg}\left(\beta - \frac{\alpha}{\beta\omega_d t_{\nu}}\right)\right] \tag{16}$$

It is clear that the formula is nonlinear, because of t_n occurring on both sides of the equation. It should be solved for every extreme iteratively. This procedure would be extremely time consuming, when solved for slightly damped systems, due to the very large number of cycles with damaging effect.

Results of performed calculations with this model have shown a weakness of the model. The term t^{α} influences the solution progressively in time. In consequence of it, the response of a blade comes never to the normal damped free vibration. This was a reason, why we tried to find a model, which expresses better such a behavior.

Model 3

When building a new model, the basic idea has come up from the fact that the influence of the internal exciting process is time-limited. Hence, the forced response should be also temporal. This requirement fulfils the following model

$$\sigma(t) = \sigma_{\beta} e^{(-\beta \pm i)\omega_d t} - \sigma_{\alpha} e^{(-\alpha \pm i)\omega_d t} \tag{17}$$

In contrary to the model 2, where $\sigma(0) = 0$, the initial stress $\sigma(0) = \sigma_{\beta} - \sigma_{\alpha}$, with σ_{α} as a magnitude of the stress excited by a change of operational parameters, and σ_{β} as a hypothetical initial magnitude of free vibration at the time $t = 0$.

The expression for evaluating times of peaks is also nonlinear and much more complicated than that of model 2. Nevertheless, the advantage of this model is the physical meaning of all quantities entering the formula, and its natural behavior as far as the form of vibrations is concerned.

Model 4

The results of experimental research of damping properties of blade materials have demonstrated (see [1], [2]) that logarithmic decrement is a function of the peak stress in the cycle. Having no better physical law, the logarithmic decrement may be expressed as a polynomial

$$\vartheta(\sigma_a) = \vartheta_o + \vartheta_1\sigma_a + \vartheta_2\sigma_a^2 + \vartheta_3\sigma_a^3 \tag{18}$$

This model of damping complies even with a linear damping with $\vartheta(\sigma_a) = \vartheta_o$. The character of damping is nonlinear in general case, as seen from eqn (18). The nonlinearity has several effects:

- the instantaneous frequency of free vibration will depend on the current amplitude,
- the sequence of extremes will not be a geometric one, what brings up that there is no closed form for the damage calculation,
- a damage should be calculated by summing elementary damages made by extremes determined by a numerical way,
- A decrease of vibration amplitude decreases also damping, what prolongs vibration, and also damage.

Since the damping is small, and amplitudes of vibration are changing very slowly, we may use the method of matrix exponential (see [3]) for the numerical integration of the equation (1) with logarithmic decrement (18). In order to do it, let us modify the original differential equation into the form

$$\ddot{\sigma}(t) + \frac{\vartheta_o}{\pi}\omega_o\dot{\sigma}(t) + \omega_o^2\sigma(t) = -\frac{1}{\pi}\sum_{\mu=1}^{3}\vartheta_\mu\sigma_a^\mu\dot{\sigma}(t) \tag{19}$$

The terms of the sum staying on the right hand side have a correction role of the linear model staying on the left hand side. As usual, the second order equation (19) is transformed into a set of two first order equations

$$\frac{d}{dt}\begin{bmatrix}\sigma(t)\\ \dot{\sigma}(t)\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_o^2 & -\dfrac{\vartheta_o\omega_o}{\pi}\end{bmatrix}\begin{bmatrix}\sigma(t)\\ \dot{\sigma}(t)\end{bmatrix} - \begin{bmatrix}0\\ \dfrac{1}{\pi}\displaystyle\sum_{\mu=1}^{3}\vartheta_\mu\sigma_a^\mu\dot{\sigma}(t)\end{bmatrix} \tag{20}$$

or more simple

$$\dot{\mathbf{s}}(t) = \mathbf{As}(t) - \boldsymbol{b}(t) \tag{21}$$

The numerical solution of the model may be found for initial conditions $\boldsymbol{s}(0) = [\sigma_o, 0]^T$ and a sampling frequency $f_s = 1/T$ in the form [3]

$$\boldsymbol{s}_n = \boldsymbol{E}_a\boldsymbol{s}_{n-1} + T\,[\boldsymbol{E}_b\boldsymbol{b}_{n-1} + \boldsymbol{E}_c(\boldsymbol{b}_n - \boldsymbol{b}_{n-1})] \tag{22}$$

There are matrices

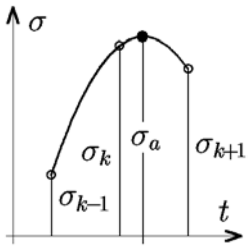
$$\boldsymbol{E}_a = \exp\boldsymbol{A}T\,, \qquad \boldsymbol{E}_b = (\boldsymbol{A}T)^{-1}(\boldsymbol{E}_a - \boldsymbol{I}), \qquad \boldsymbol{E}_c = (\boldsymbol{A}T)^{-1}(\boldsymbol{E}_b - \boldsymbol{I}) \tag{23}$$

Taking into account the form of the vector $\boldsymbol{b}$, the equation (22) may be rewritten into more effective form.

The form of the correction vector $\boldsymbol{b}$ is rather unpleasant, because of the presence of the immediate amplitude σ_a. It means that the nonlinear part of damping depends on an envelope to the transition vibration, which is not at disposal during the solution of the differential equation. It has to be found by another way.

3. Damage Calculation

It has already been said that a damage may easily be obtained for model 1 only, since the closed form for it is known. For all other models we may obtain only samples of the damped process by calculation or by a measurement. The samples do not express extreme values with a high accuracy unless the sampling frequency is very high, what would be very inefficient. The following process may compensate the disadvantage of big errors introduced by the sparse sampling:



The course of a stress in time may be fitted by a harmonic function in a short time interval. It is possible to calculate an approximate value of the extreme out of the biggest value of samples and two values of neighboring samples using the formula

$$\sigma_{a\nu} = \sigma_k\sqrt{\frac{\sigma_k^2 - \sigma_{k-1}\sigma_{k+1}}{\sigma_k^2 - \left(\dfrac{\sigma_{k-1} + \sigma_{k+1}}{2}\right)^2}} \tag{24}$$

derived with the use of Z transform (see [4], [5]).

The total damage produced by a single transient process is obtained as a sum of all elementary damages defined by the equation (10), which now takes the form

$$d_\nu = \frac{1}{N_c\sigma_c^w}\sum_{\nu=0}^{n-1}\sigma_{a\nu}^w\,, \tag{25}$$

with number n of all amplitudes $\sigma_{a\nu} \geq \sigma_c$.

The described procedure has been applied to models 1-3 with artificial material data. There was no difference between damages obtained via eqn (13) and eqn (25) with approximated extremes due to eqn (24) for model 1. Transient processes of models 1-3 calculated with material data $\sigma_c = 300$ MPa and $w = 4$

are plotted in the figure 2. The magnitudes of initial stress were $\sigma_o = \sigma_\beta = 540$ MPa, and $\sigma_\alpha = 250$ MPa. Model 2 gives completely different shape of the process, while models 1 and 3 are similar for longer times.

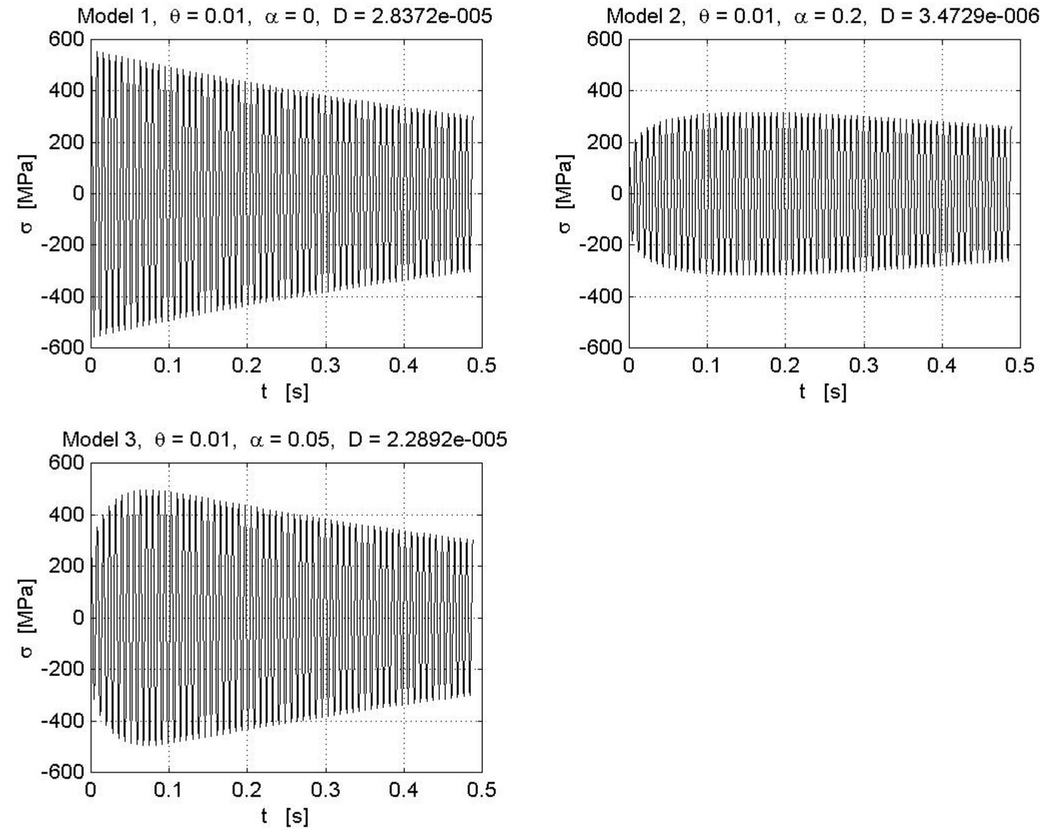


Figure 2: Stress responses of models

The results were obtained by running a program, which enables to analyze both three above mentioned models of transients and even a file with samples of a real damped vibration process measured in operational conditions. The program is built in MATLAB v. 5.3. More to it, the program analyzed also an influence of a variety of logarithmic decrement ϑ and the power w of S-N curve for the model 1. The results of this analysis are presented in the figure 3.

The figure 3 enables to get a good insight into the influence of the logarithmic decrement ϑ on the number of damaging stress cycles and in connection with the power w on the total damage D caused by one realization of the transient process. The number N of repetition of the process to the break of the blade under the above given material parameters and σ_o will be

$$N = \frac{k_{PM}}{D} \tag{26}$$

The coefficient k_{PM} involves a difference between the reality and the accepted hypothesis of linear cumulation of damage due to Palmgren-Miner.

4. Conclusions

The presented results show how complicated is the appropriate choice of the transient model for blades in unsteady conditions. The identification of a proper model of transients and an identification of its parameters have to be selected in the near future. The investigations will be also oriented to the identification of nonlinear damping parameters.

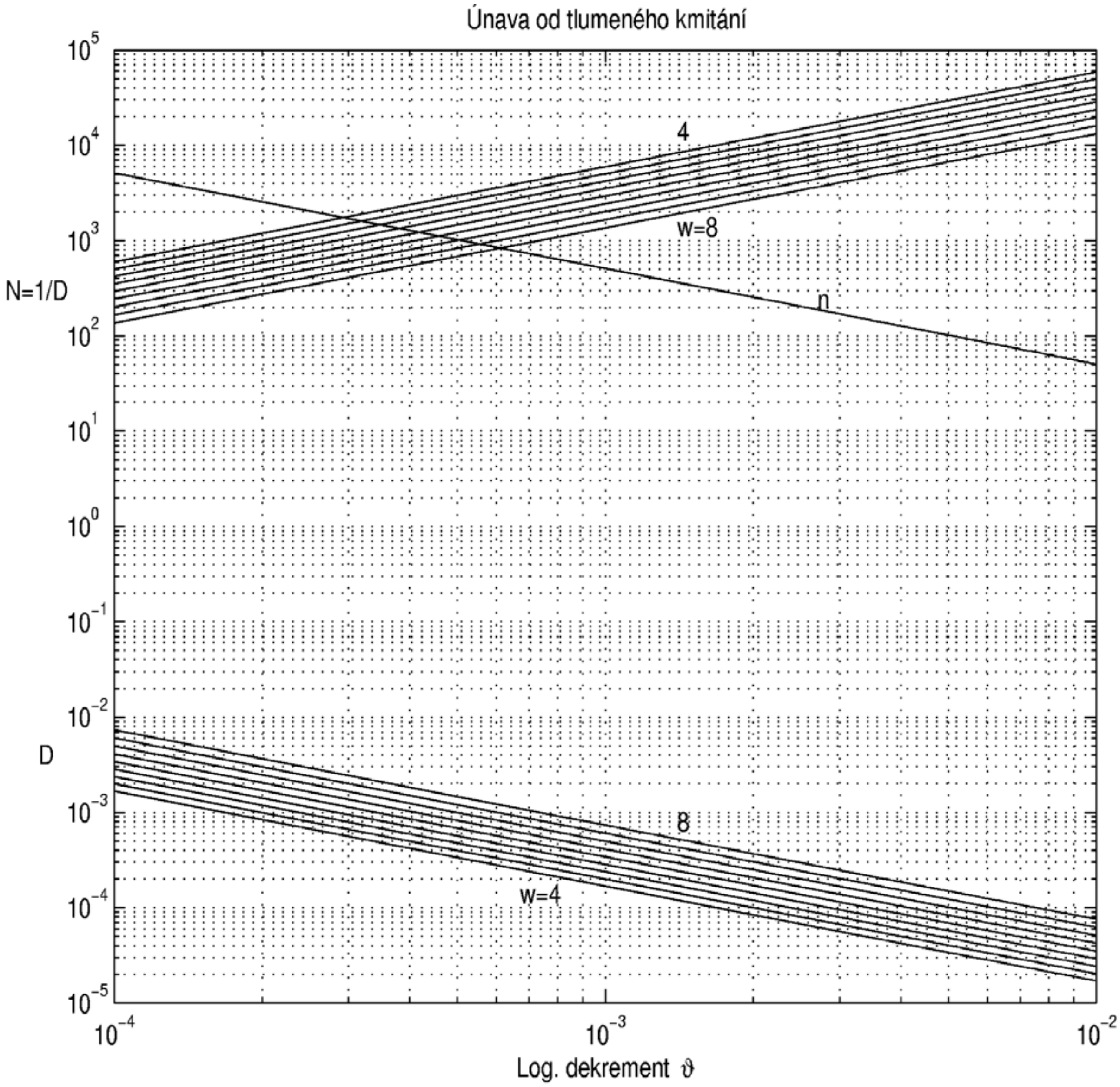


Figure 3.: The influence of ϑ ond n, D, and N .

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