Prediction of Damage Cumulation in Vibrating Rotors

M. Balda
Institute of Physical Engineering, University of West Bohemia
and Academy of Sciences of the Czech Republic
Veleslavínova 11, 301 14 Pilsen, Czech Republic

Abstract

A problem of fatigue cracks is very important. The paper deals with estimating an amount of damage caused by varying generalized forces in vibrating rotors. An approach to optimization of rotors with guaranteed fatigue life is briefly discussed.

Nomenclature

β	"effective" stress concentration factor
$c_x,\ c_y$	exponents of regression functions
$\boldsymbol{C}$	matrix of regression coefficients
d	smaller diameter in a notch
d_i	elementar damage caused by one closed cycle
D	bigger diameter nearby a notch
D_o	total damage raised within time of observation T_o
E	Young's modulus
$f(\boldsymbol{p})$	criterion (goal) function to be optimized
φ	phase angle
G	shear modulus
$\boldsymbol{g}(\boldsymbol{p})$	vector of inequality constraints
$\boldsymbol{h}(\boldsymbol{p})$	vector of equality constraints
$L(\boldsymbol{p})$	service life of a rotor
J	quadratic moment of a cross-section
k_d	service life coefficient
k_q	surface quality factor
k_V	part size factor
K_t	stress concentration factor

M	moment
N	number of harmonic cycles to fracture
$\boldsymbol{p}$	vector of design parameters
ϱ	radius of a notch
R_m	strength of a material
s	stress σ or τ
$s_a,\ s_m$	amplitude and mean stress respectively
t	time
T_o	time of observation
$v,\ w$	vertical and horizontal deflections
ξ	$(D-d)/D$, relative narrowing of a size in a notch
η	ϱ/D, relative radius of a notch
$\boldsymbol{x},\ \boldsymbol{y}$	vectors of regression functions

1. Introduction

Rotors running in real bearings exhibit rather complicated spatial movement, when rotating, what was shown already many years ago (see [1], [2]). A numerical analysis of such problem gives a solution, which containes not only deformations, but also their derivatives. It is quite easy to evaluate courses of bending and shear stresses along a shaft with the use of the solution, material modulus and a diameter. The stresses and their variations in time may serve for a damage estimation, which they caused.

2. Causes of fatigue crack initiation

Stress varying in time may exhaust a resistance of the rotor material against damage, and generate microscopic plastic deformations in the vicinity of material imperfections. In such disturbed area, microcracks may be initiated, which may come to a macrocrack by mutual interconnections. Provided the macrocrack is propagating, it becomes dangerous. This is the reason why the problem of its identification is pursued both in literature and conferences. Hence, it is important to know the causes of the fatigue crack initiation in rotors, and to design rotors with the minimized danger of those cracks, at least in the guaranteed period of their service lifes.

Locations of cracks may not be stated in forward with the full certainty. However, they are as usual in places with highest dynamic stresses, so-called critical points. Those are locations with serious material imperfections, and places possessing severe geometrical changes of a cross-sections of a shaft like in grooves and shoulders. A critical point may also lay at the edge of a shrunk-on wheel or a flange coupling.

Even that rotors are loaded by a combined stress, it is quite general that one component of a complex stress is dominant. For sake of simplicity, it is convenient to investigate different types of dynamic stresses separately.

2.1. Bending of shafts

A bending stress is generated by an arbitrary lateral shaft deflection. The reason of the deflection are generalized forces. Provided the forces are rotating in a fixed space with a frequency ω_f, and the rotor with ω_r, a stress they cause on a surface of the rotor rotates with the relative frequency

$$\omega_\sigma = \omega_f - \omega_r \tag{2.1}$$

or its multiple. The forces may have a different character:

2.1.1 Forces fixed in space

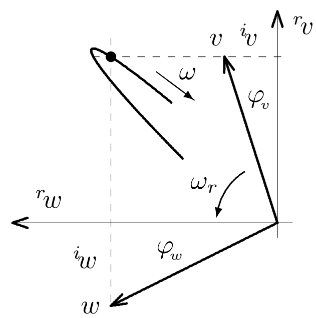
All forces comming from gravitation, partial admission of steam, magnetic fields, and forces acting in tooth-, belt- and chain transmissions belong to this group. If they are constant, they produce a stress traveling round a circumference of the shaft. The frequency of changes in every point of the surface is $\omega_\sigma = 0 - \omega_r = -\omega_r$. The nominal quantity of the stress on the shaft surface is

$$\sigma(x) = \frac{32\, M_b(x)}{\pi\, d^3(x)} = -\frac{32\, E\, J(x)}{\pi\, d^3(x)}\, \frac{\partial^2 y}{\partial x^2} \tag{2.2}$$

where $y(x)$ is the maximum deflection of the shaft in a distance x from an origin. The forces might initiate a circumferential crack propagating from the surface of the isotropic shaft to its center. Its identification out of measured data would be rather difficult, since it does not generate changes in twice-per-revolution component of vibrations. This does not hold for nonisotropic rotor. The forces might generate one crack or a couple of oposite cracks, which would be identifiable out of vibration data containing a $2\,\omega_r$ component.

2.1.2 Out of balance forces

In this case of deterministic excitation, the frequency of unbalanced forces is $\omega_f = \omega_r$, and hence the resulting frequency of a stress equals zero due to equation (2.1). The forces would cause a static rotor deflection rotating by the equal frequency with the rotor. However, this situation takes place only in case of isotropic bearings, when a trajectory of any point of the rotor axis is circular. If the rotor is carried by general anisotropic bearings, the trajectories are no more circles.



Let a point of the rotor axis vibrates in a lateral plane. The plane is defined by real vertical axis ^rv, and horizontal axis ^rw. Let amplitudes and phases of the vibration be $|v|, \varphi_v$ and $|w|, \varphi_w$ respectively. The coordinates of a point of the trajectory

$$\begin{aligned} \begin{bmatrix} ^rv \\ ^rw \end{bmatrix} &= \mathrm{Re} \begin{bmatrix} |v|\, e^{i\varphi_v} \\ |w|\, e^{i\varphi_w} \end{bmatrix} e^{i\omega t} \\ &= \begin{bmatrix} |v|\, \cos(\omega_r t + \varphi_v) \\ |w|\, \cos(\omega_r t + \varphi_w) \end{bmatrix} \end{aligned} \tag{2.3}$$

are yielding the instantaneous distance of the trajectory from the equilibrium point

$$y = \sqrt{^rv^2 + {^rw}^2} \tag{2.4}$$

The equation of the trajectory may be easily derived from the equation (2.4) in the form

$$\left(\frac{^rv}{|v|}\right)^2 - 2\,\frac{^rv}{|v|}\frac{^rw}{|w|}\, \cos\Delta\varphi + \left(\frac{^rw}{|w|}\right)^2 = \sin^2\Delta\varphi\ , \tag{2.5}$$

where $\Delta\varphi = \varphi_v - \varphi_w$. The equation (2.5) is a quadratic form (see [3]) expressing an ellipse with skewed axes. An analysis of the quadratic form gives formula for lengths of ellipse half-axes

$$a_j = \left| \frac{\sin \Delta\varphi}{\sqrt{c_j}} \right| , \qquad j = 1,\ 2\ , \tag{2.6}$$

where

$$c_j = \frac{1}{2} \left(\frac{1}{|v|^2} + \frac{1}{|w|^2} \right) \left\{ 1 \pm \left[1 - \frac{4\,\sin^2 \Delta\varphi}{\left(\left| \frac{v}{w} \right| + \left| \frac{w}{v} \right| \right)^2} \right]^{\frac{1}{2}} \right\} \tag{2.7}$$

The angle of the first axis is

$$\alpha_1 = \arctan \frac{1 - |v|^2 c_1}{\left| \frac{v}{w} \right| \, \cos \Delta\varphi} \tag{2.8}$$

It may be proved that the point circulates on the trajectory with the same sign as ω, if $\sin \Delta\varphi$ were positive. Hence, the ellipse degenerates into a single straight-line segment for $\Delta\varphi = 0,$ or π.

Changes of stress depend on changes of shaft deformation. Frequency of the changes is now $\omega_\sigma = 2\,\omega_r$, since two minima and two maxima occur during one turn of the trajectory. A quantity of nominal stress changes may be determined through the same formula (2.2), since bending moments depend on shaft deformations under the same law.

A crack initiation might be expected only in case of extra high levels of shaft vibration. The crack would propagate from one point on the surface of the rotor. It might be well identified out of twice-per-revolution component of vibrations.

2.1.3 The other forces

The group of these forces is composed of forces generated by oil-film, steam and other media, and similar, when operating in a range of self-excited vibrations or in chaos regime. They generate complex deformations, and consenquently even stresses. Only in case of harmonic vibration, the frequency of stress variations in certain point is given by formula (2.1). Their values should be either measured or guessed basing on the assumption of shaft deflections exhausting all the bearing clearences. Should a crack be initiated, it would have concentric shape. An identification of it would be difficult.

2.2. Torsion of shafts

It is well known that nominal torsional stress reaches its highest values on a surface of the shaft

$$\tau(x) = \frac{16\, M_t(x)}{\pi\, d^3(x)} = -\frac{G J_p(x)}{\pi\, d^3(x)} \frac{\partial \alpha_x}{\partial x} \tag{2.9}$$

Changes of the stress are slow if loading of the machine varies slowly. A different situation takes place, if the load changes suddenly. At that time, inertial masses influence a twist α_x of the shaft, which starts to vibrate. Transitions in the form of free torsional vibration take place, when the load is switched off. Much stronger torsional vibration is generated during a short circuit on generator terminals. The vibration is excited by a short-circuit torque. Its course is complicated, and depends both on intensity of the torque and on dynamic properties of the turboset rotors.

A crack, which might be initiated by this type of loading, has a concentric shape. It propagates from the surface toward the center of a shaft. Its identification is difficult.

3. A damage and its cumulation

Fatigue cracks generated in large rotors by varying stress are very dangerous. Hence, it is necessary to have both tools for estimating an amount of a damage caused by known time functions of a stress, and a collection of material data containing values of critical amplitudes of a stress as a function of a mean stress s_m, $s_{cm} = s_c(s_m)$, and corresponding number of harmonic cycles $N_{cm} = N_c(s_m)$. Symbol s substitutes both normal stress σ and tangential stress τ. Note that $s_{c0} = s_c(0) = s_c$ is a standard fatigue limit of the material, and $N_{c0} = N_c(0) = N_c$ a corresponding number of cycles to a fracture.

3.1. Generalized Haigh's plot

The function $s_c(s_m)$ is well known as Haigh's plot, which gives the information on critical amplitudes of stress as a function of the mean stress.

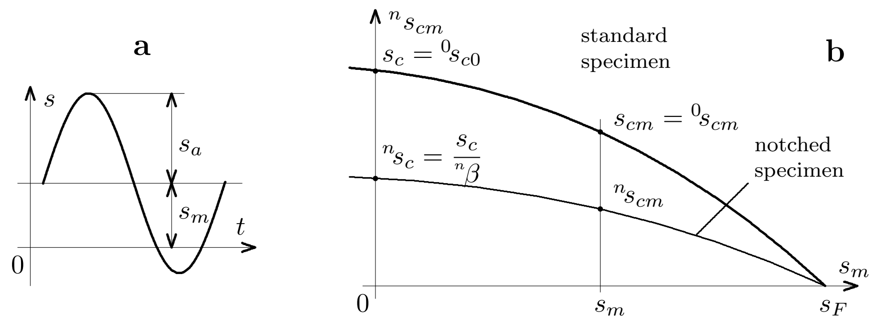


Figure 3-2: Harmonic stress cycle (a) and generalized Haigh's plot (b)

Let us denote a critical amplitude s_a of a nominal stress variation about a mean stress s_m in a notch as ${}^n s_{cm}$ where superscript n denotes the type of the notch and a loading. There are many formulae, which approximate the function ${}^n s_{cm}$ in the literature. Let it hold for a general curve of the plot

$$ {}^n s_{cm} = \frac{s_c}{{}^n\beta} \left(1 - \frac{s_m}{s_F}\right)^{k_H} = \frac{s_{cm}}{{}^n\beta} \tag{3.1} $$

The coefficient ${}^n\beta = \dfrac{s_{cm}}{{}^n s_{cm}}$ is an effective stress concentration factor of a given notch, a kind of loading, and a material of the specimen. There are two more material constants in the formula, k_H and s_F. The later one used to be expressed as a multiple of a material strength R_m.

The coefficient ${}^n\beta$ of a rotor is either obtained experimentally, or estimated out of a theoretical stress concentration factor K_t as a function ${}^n\beta = {}^n\beta({}^n K_t,\ k_V,\ k_q)$ in the form

$$ {}^n\beta = \frac{\beta\left({}^n K_t\right)}{k_V\, k_q}\ , \tag{3.2} $$

where k_V is a coefficient depending on a volume of the material near by crack, and k_q is a coefficient which takes into account a quality of the surface. There is a plenty of formulae for an effective stress concentration factor $\beta\left({}^n K_t\right)$. Neuber's formula [4] is

$$ \beta\left({}^n K_t\right) = 1 + \frac{{}^n K_t - 1}{1 + \sqrt{\frac{A}{\varrho}}} \tag{3.3} $$

where $A(R_m)$ [mm] is Neuber's coefficient dependent on the strength R_m expressing a size of material grains, and ϱ [mm] is a radius of curvature in the notch root. The following formula may serve for practical purposes:

$$\sqrt{A} = 289.1/R_m - 0.1217 \tag{3.4}$$

3.2. Theoretical stress concentration factor

The theoretical stress concentration factor ${}^{n}K_t(\xi, \eta)$ depends on a form of a notch, kinds of a loading, a relative narrowing ξ of a size by the notch, and its relative sharpness η, where

$$\xi = \frac{D-d}{D}, \quad x = \xi^{{}^{n}c_x} \qquad \text{and} \qquad \eta = \frac{\varrho}{D}, \quad y = \eta^{{}^{n}c_y} \tag{3.5}$$

The theoretical stress concentration factor ${}^{n}K_t$ may be approximated by the bilinear form

$${}^{n}K_t = 1 + \boldsymbol{x}_{K}^{T}\, {}^{n}\boldsymbol{C}_{K}\, \boldsymbol{y}_{K} \ , \tag{3.6}$$

where

$$\begin{array}{lcl} \boldsymbol{x}_{K} & = & [\, p_1(x),\ p_2(x),\ ...,\ p_5(x) \,]^T \ , \\ \boldsymbol{y}_{K} & = & \left[\, y^{-2},\ y^{-3},\ y^{-4},\ y^{-5} \,\right]^T \ , \\ {}^{n}\boldsymbol{C}_{K} & \in & \mathcal{R}^{5,4} \end{array} \tag{3.7}$$

The elements of the vextor $\boldsymbol{x}_{K}$ are values of orthogonal polynomials with roots at the ends of the interval $x \in \langle 0,\ 1 \rangle$. Their coefficients are presented in the **Tab. 3-1**:

n	$p_1(x)$	$p_2(x)$	$p_3(x)$	$p_4(x)$	$p_5(x)$	$p_6(x)$
7	0	0	0	0	0	95.3333
6	0	0	0	0	-33.0000	-333.6667
5	0	0	0	12.0000	99.0000	458.3333
4	0	0	-4.6667	-30.0000	-111.0000	-311.6667
3	0	2.0000	9.3333	26.0000	57.0000	108.3333
2	-1.0000	-3.0000	-5.6667	-9.0000	-13.0000	-17.6667
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0	0	0	0	0	0	0

Table 3-1: Coefficients of orthogonal polynomials

The matrix ${}^{n}\boldsymbol{C}_{K}$ and coefficients ${}^{n}c_x$ and ${}^{n}c_y$ were found by minimizing the maximum absolute value of a difference of the formula (3.6) from exact values of ${}^{n}K_t$, which have been published in [5] – [7] for 17 different types of notches and kinds of loads. In the following table, there are the necessary data for types n, which are relevant for rotors.

n	type	$\max\lvert\varepsilon_{ij}\rvert$ [%]
RVB	**R**ound – **V**-notch – **B**ending	0.4
RVT	**R**ound – **V**-notch – **T**orsion	0.5
RSB	**R**ound – **S**houlder – **B**ending	0.9
RST	**R**ound – **S**houlder – **T**orsion	0.3

The next structure contains data, which are arranged in the following way:

$$
{}^{n}\boldsymbol{C}_{ij} = \begin{bmatrix} & {}^{n}\boldsymbol{C}_{K} \in \mathcal{R}^{5,4} & & \\ {}^{n}c_x & {}^{n}c_y & \xi_{min} & \xi_{max} \\ \max\lvert\varepsilon_{ij}\rvert & \mathrm{RMS}\,(\varepsilon_{ij}) & \eta_{min} & \eta_{max} \end{bmatrix} \tag{3.8}
$$

```
Cij=[...
% RVB  (Round, V-notch, Bending)  % RSB  (Round, shoulder, Bending)
 -0.9546  1.7172 -0.3811    0.0290  0.3990  -0.0233  -0.0051  0.0006
 -0.6711  1.9695 -0.9084    0.1291  0.5843  -0.2283   0.0340  -0.0018
  1.2226 -0.9075  0.3545  -0.0370   0.6924  -0.3829   0.0754  -0.0048
  1.2147 -1.0873  0.3996  -0.0577   0.1309  -0.0159  -0.0049  0.0006
  0.6010 -0.4576  0.1057    0.0017 -0.1014    0.1382  -0.0344 0.0025
  0.6395  0.2480  0.0200    0.9000  0.6785    0.4655  0.0500  0.9000
  0.4110  0.2253  0.0150    0.5000  0.9300    0.3981  0.0150  0.5000
% RVT  (Round, V-notch, Torque)   % RST  (Round, shoulder, Torque)
 -0.5424  0.7371  0.0546  -0.0347   5.7744 -13.9186  10.1701  -1.8458
 -0.5394  1.2792 -0.5787    0.0848 -5.6001  11.5741  -7.3316  1.5014
  0.7658 -0.8293  0.4064  -0.0473  -5.0089  13.3690 -11.6788  3.4447
 -0.1544  0.7249 -0.4469    0.0733 -3.0636    6.2668  -3.6547 0.5428
 -0.0410  0.4526 -0.3409    0.0772 14.1124 -30.7907  21.8907  -5.0485
  0.6601  0.2098  0.0200    0.9000  0.6322    0.1044  0.0500  0.9000
  0.4909  0.2172  0.0100    0.5000  0.3078    0.1049  0.0150  0.5000]
```

3.3. A damage estimation

A relative damage of the rotor may be estimated under a series of hypotheses. The hypothesis of linear cumulation of the damage due to Palmgren - Miner has proved to be appropriate, when applied to full stress cycles, which correspond to closed hysteretic loops in the stress-strain plot. For the purpose, the time histories of stress records or recalculated to critical points are analyzed by the multichannel "rain-flow" method, which decomposes complex signals into closed cycles (see [8], [9]).

An elementar relative damage of round specimen may be determined using the equation of an Whler's (S-N) curve

$$\frac{N_a}{N_c} = \left(\frac{s_c}{s_a}\right)^w \tag{3.9}$$

After generalization of this equation for critical point of the rotor with a concentrator, i-th full stress cycle of a nominal amplitude s_{ai} and mean stress s_{mi} will cause a damage

$$^n d_i = \frac{1}{^n N_{ai}} = \frac{1}{^n N_c} \left(\frac{s_{ai}}{^n s_{cmi}}\right)^{^n w} \tag{3.10}$$

The total relative damage caused during a time of observation T_o should be

$$^n D_o = \sum_i {}^n d_i \leq k_d \tag{3.11}$$

for save run of a machine. If the loading process be a characteristic sequence, the service life of the rotor would be

$$^n L = k_d \, \frac{T_o}{^n D_o} \;, \tag{3.12}$$

where the coefficient k_d equals 1, provided that the law of linear cumulation of damage holds exactly. Otherwise, its value should be found by experiment.

4. Optimization of a rotor design

There are many possibilities how to optimize the design of the rotor. The following approach comprises several points of view on the optimality. The rotor design may be declared as optimum one, if it fulfils all guaranteed properties and is cheapest. It is difficult to build a criterion function based on the product price. However, the price is strongly dependent on the mass of the rotor. Hence, the criterion function may be just a mass of the rotor m_r. The guaranteed service life L_G of the rotor and other technological requirements may be formulated as constraints. Then the problem sounds

$$
\begin{array}{llcl}
\text{minimize} & f(\boldsymbol{p}) & = & m_r(\boldsymbol{p}) \\
\text{by constraints} & L(\boldsymbol{p}) & \geq & k_d\, L_G(\boldsymbol{p}) \\
 & \boldsymbol{g}(\boldsymbol{p}) & \geq & \mathbf{0} \\
 & \boldsymbol{h}(\boldsymbol{p}) & = & \mathbf{0}
\end{array}
\tag{4.1}
$$

Vector $\boldsymbol{p}$ contains a set of optimization parameters. It is clear that the requirement concerning the service life $L(\boldsymbol{p})$ is only a special constraint of the inequality type $\boldsymbol{g}(\boldsymbol{p})$.

It is neccessary to investigate a set of critical points during optimization, because the optimization process may change the location of the most critical place. This requires an application of the multichannel version of the rain-flow method (see [10]), and an optimization procedure for minimizing nonsmooth goal functions.

5. Conclusions

Prediction of a damage cumulation is rather delicate procedure just like a consequential prediction of a service life of the rotor. High indeterminacy of the result is a consequence of large variances of material properties and other technological quantities, inexact knowledge of the excitation, and a simplification of multiaxial stress into uniaxial one. If the later assumption were false, the problem would become of an order more complicated and not solved to full satisfaction yet. However, the presented approach may serve as the first attempt to design reliable rotors optimal both in price and service.

It has to be stressed out that the described approach links up dynamics of rotors, their reliability, and the minimum costs of a product into the unique process. Optimization procedure applied to mass criterion tends to reduce of rotor diameters, what changes both dynamic properties and resistence against fatigue crack initiation in critical points, which may alternate during optimization. A set of constraints ensures that the final design fulfils all requirements, guaranteed service life as well. Presented formulae allow to include stress concentration factors and evaluation of the rotor fatigue life in the optimization process.

The similar approach may be chosen even for blading. If a mesurement system were installed on a turboset for monitoring blade vibrations, the detailed analysis of measured data could give valuable information on a residual length of the service life.

Acknowledgment

The contribution has been prepared under the support of the Grant Agency of the Czech Republik under contracts 101/95/0087 and 101/97/0226.

References

[1] M. Balda: Dynamic Properties of Turboset Rotors. In: Dynamics of Rotors, Proc. Symp. IUTAM, Lyngby 1974, ed. F. I. Niordson, Springer, Berlin, 1975

[2] R. Gasch, H. Pftzner: Rotordynamik – Eine Einfhrung. Springer, Berlin, 1975

[3] B. Noble, J. W. Daniel: Applied Linear Algebra. Prentice-Hall, Englewood Cliffs, 1977

[4] H. Neuber: Kerbspannungslehre. Springer, Berlin, 1937

[5] H. Nisitani, N. A. Noda: Stress concentration of a cylindrical bar with a V-shaped circumferential groove under torsion, tension or bending. Eng. Fracture Mech., Vol. 20, No 5/6, pp. 743-766, 1984

[6] N. A. Noda, M. Sera, Y. Takase: Stress concentration factors for round and flat specimens with notches. Int. J. Fatigue, Vol. 17, No. 3, pp. 163-178, 1995

[7] N. A. Noda, Y. Takase, K. Monda: Stress concentration factors for shoulder filets in round and flat bars under various loads. Int. J. Fatigue, Vol. 19, No 1, pp. 75-84, 1997

[8] S. D. Downing, D. F. Socie: Simple Rainflow Counting Algorithms. Int. Jour. of Fatigue, Vol. 4., No 1, pp. 1-31, 1982

[9] M. Balda: Digital processing of extremes of random processes (In Czech). Research report, SKODA Works, Central Research Institute, Pilsen, 1976

[10] M. Balda: Multichannel tracing a cummulation of damage in real time (in Czech). Proc. of Symp. Dynamics of Machines '96, Inst. of Thermomechanics of the Czech Academy of Sci, Prague, 1996